SEMESTER TWO 2017

YEAR 12, Units 3 & 4

MATHEMATICS METHODS

Section One – Booklet 1
(Calculator–free)

Name: Marking Key

Circle your Teacher: **MAW** **VMU** **MPC** **AGC**

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: five minutes
Working time for section: fifty minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

IMPORTANT NOTE TO CANDIDATES
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.
Formula sheet which may also be used for Section Two.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	**9**	**9**	**50**	**51**	**35**
Section Two Calculator—assumed	12	12	100	87	65
Total marks				138	

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017.* Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

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Question 1 **(3+3+2+2 = 10 marks)**

Differentiate, simplifying and leaving your answers with positive indices where appropriate:

a) $y = \dfrac{x^2 - 4x}{e^{2x}}$

$$\frac{dy}{dx} = \frac{(2x-4)e^{2x} - (x^2-4x)(2e^{2x})}{(e^{2x})^2}$$ ✓ correct numerator ✓ correct denominator

$$= \frac{2e^{2x}\left((x-2) - (x^2-4x)\right)}{(e^{2x})^2}$$

$\boxed{3}$

$$= \frac{2\cancel{e^{2x}}(-x^2+5x-2)}{(e^{2x})^{\cancel{2}}}$$

$$= \frac{-2(x^2-5x+2)}{e^{2x}}$$ ✓ simplified answer

b) $g(x) = 2x\cos(e^{2x})$

$$g'(x) = 2\cos(e^{2x}) + 2x(2e^{2x})(-\sin(e^{2x}))$$ ✓ ✓ correct application of the product rule.

$$= 2\cos(e^{2x}) - 4xe^{2x}(\sin(e^{2x}))$$ ✓ simplification $\boxed{3}$

Question 1 continued

c) $y = \log x$

$y = \dfrac{\ln x}{\ln 10}$ ✓ rewrites

$\boxed{2}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{x \cdot \ln 10}$ ✓ correct derivative

d) $G(x) = \int_1^{3x} \sin^2(1+e^t)\,dt$

$G'(x) = \dfrac{d}{dx} \int_1^{3x} \sin^2(1+e^t)\,dt$

$= 3\sin^2(1+e^{3x})$

✓ coefficient of 3

✓ $\sin^2(1+e^{3x})$

$\boxed{2}$

Question 2 **(3+3 = 6 marks)**

Use calculus to determine the following indefinite integral.

a) $\int \left(\frac{2x+3}{3x+x^2} \right) dx$

$\int \left(\frac{2x+3}{x^2+3x} \right) dx$

$= \ln |x^2+3x| + c$

✓ $\ln (x^2+3x)$

✓ Absolute value of x^2+3x

✓ $+ c$

$\boxed{3}$

Use Calculus to determine the exact value of each of the following.

b) $\int_1^4 \frac{\sqrt{x}+x^3}{x^2} dx$

$= \int_1^4 \left(\frac{\sqrt{x}}{x^2} + \frac{x^3}{x^2} \right) dx$

$= \int_1^4 \left(x^{-\frac{3}{2}} + x \right) dx$ ✓ simplification

$= \left(-2x^{-\frac{1}{2}} + \frac{1}{2}x^2 \right) \Big]_1^4$ ✓correct integration

$\boxed{3}$

$= \left(-2(4)^{-\frac{1}{2}} + \frac{1}{2}(4)^2\right) - \left(2(1)^{-\frac{1}{2}} + \frac{1}{2}(1)^2\right)$

$= (-1 + 8) - (-2 + \frac{1}{2})$

$= 8\frac{1}{2}$ or $\frac{17}{2}$ ✓correct evaluation.

Question 3 **(2+3 = 5 marks)**

a) Determine $\frac{d}{dx}(e^x \ln x^2)$

$= e^x . \left(\frac{2}{x}\right) + e^x . \ln x^2$ ✓ ✓

correct use of product rule.

$\boxed{2}$

b) Hence or otherwise, calculate the exact value of $\int_1^2 \frac{e^x}{x}\left(2 + x\ln x^2\right)dx$

Since $\frac{d}{dx}(e^x \ln x^2)$

$= e^x . \left(\frac{2}{x}\right) + e^x . \ln x^2$

$= e^x\left(\frac{2}{x} + \ln x^2\right)$

$= \frac{e^x}{x}\left(2 + x . \ln x^2\right)$ ... So $\int_1^2 \frac{e^x}{x}\left(2 + x\ln x^2\right)dx$

✓ factored form

$= \left(e^x . \ln x^2\right)\Big]_1^2$ ✓ correct integration

$= e^2 . \ln 4 - e^1 . \ln 1$

$= e^2 . \ln 4$ ✓ correct evaluation

$\boxed{3}$

Question 4 **(4+2 = 6 marks)**

The discrete random variable X represents the outcome on a spinner. The probability distribution of X is displayed in the table below.

x	0	1	2	3	4
$P(X=x)$	$2n$	n	m	m	0.1

a) Given that E(X) = 2 determine the values of m and n.

$2n+n+m+m+0.1=1$
$3n+2m=0.9 \quad — ①$ ✓

$0\times 2n+1\times n+2\times m+3\times m+4\times 0.1=2$
$n+5m=1.6 \quad — ②$ ✓
$n=1.6-5m$

Substitute $n=1.6-5m$ into $3n+2m=0.9$
$3(1.6-5m)+2m=0.9$
$4.8-15m+2m=0.9$
$-13m=-3.9$
$m=0.3$

Substitute $m=0.3$ into $n=1.6-5m$
$=1.6-1.5$
$=0.1$

$\therefore m=0.3$ ✓ and $n=0.1$ ✓

$\boxed{4}$

b) E(X^2) = 5.6 determine the value for Var(Z), where $Z = 10X - 7$

$Var(X) = E(X^2) - \mu^2$
$= 5.6 - 2^2$ ✓ calculates Var(x)
$= 1.6$

$\therefore Var(Z) = 10^2 \times 1.6$ ✓ concept $Var(Z) = 10^2 \times Var(X)$
$= 160.$

$\boxed{2}$

Question 5 **(2+3 = 5 marks)**

Given that the $\log_5 2 = p$ and the $\log_5 9 = k$, express each of the following in terms of p and k.

a) $\log_5 36 = \log_5(4\times 9)$

$= \log_5 4 + \log_5 9$

$= 2\log_5 2 + \log_5 9$ ✓ correct use of log laws.

$= 2p + k$ ✓ Correct substitution.

$\boxed{2}$

b) $\log_5(0.9) = \log_5\left(\frac{9}{10}\right)$

$= \log_5 9 - \log_5 10$ ✓ correct use of log laws

$= \log_5 9 - \log_5(2\times 5)$

$= \log_5 9 - [\log_5 2 + \log_5 5]$ ✓ correct use of log laws

$= k - [p + 1]$

$= k - p - 1$

substitution ✓

$\boxed{3}$

Question 6 **(4+3 = 7 marks)**

a) Solve $3[\log_3(x)]^2 - 28\log_3(x) + 9 = 0$, giving **exact answer(s)**.

Let $y = \log_3 x$

$\therefore 3y^2 - 28y + 9 = 0$

$\therefore (3y-1)(y-9) = 0$ ✓ correctly factorises quadratic

$y = \frac{1}{3}$ or $y = 9$ ✓ solves factors

$\therefore \log_3 x = \frac{1}{3}$ or $\log_3 x = 9$

$\therefore x = 3^{\frac{1}{3}}$ ✓ or $x = 3^9$ ✓ Solutions to question.

$\boxed{4}$

b) Express y in terms of x if $\ln(2x) + 2 = \dfrac{\ln(5y)}{3}$, simplify your answer.

$\ln(2x) + 2 = \dfrac{\ln(5y)}{3}$

$\therefore e^{\ln(2x)+2} = e^{\frac{\ln(5y)}{3}}$ ✓ raise each side ^ e.

$\therefore 2x \cdot e^2 = (5y)^{\frac{1}{3}}$ ✓ simplifies

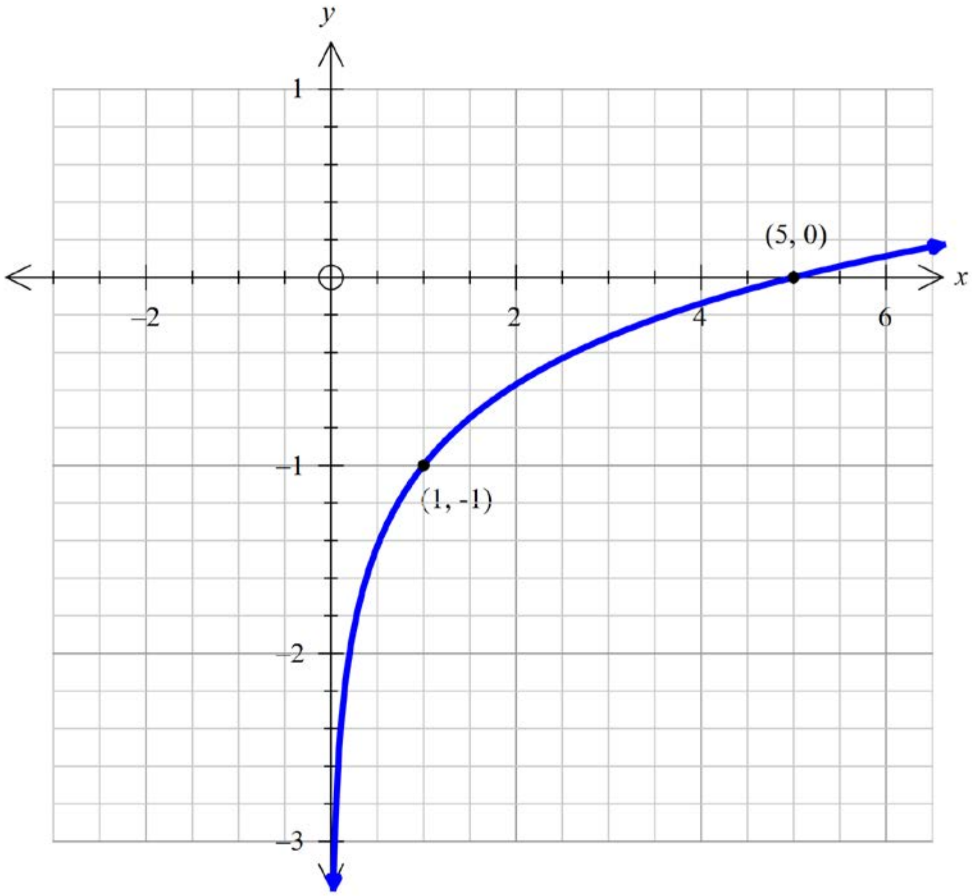
$\therefore 8x^3e^6 = 5y$

$\therefore y = \dfrac{8e^6x^3}{5}$ ✓ rearrange to get $y = \ldots$

$\boxed{3}$

Question 7 **(2 marks)**

Determine the equation of the graph shown below.



$y = \log_5 x - 1$ ✓ ✓

$\boxed{2}$

Question 8 **(3 marks)**

Determine the exact value of the x-intercept for the function defined by $y = 3e^{\left(-\frac{1}{2}x\right)} - 2$

x-intercept occurs when $y=0$
Solve $3e^{-\frac{1}{2}x} = 2$ ✓ solving for $y=0$

$e^{-\frac{1}{2}x} = \frac{2}{3}$

$\ln\left(e^{-\frac{1}{2}x}\right) = \ln\left(\frac{2}{3}\right)$ ✓ ln of both sides

$-\frac{1}{2}x = \ln\left(\frac{2}{3}\right)$

$x = -2\ln\left(\frac{2}{3}\right)$ ✓ solves for x

$\boxed{3}$

Question 9 **(6 marks)**

A random variable X has a mean of $\frac{4}{3}$ and a probability density function:

$f(x) = \frac{x}{k}$ for $0 \le x \le a$, determine the values of k and a.

$$\therefore \int_0^a \left(\frac{x}{k}\right) dx = \frac{1}{k}\int_0^a x\,dx \checkmark$$

$$= \frac{1}{k}\left(\frac{x^2}{2}\right)\Big]_0^a$$

$$= \frac{a^2}{2k}.$$

But $1 = \frac{a^2}{2k}$ ✓

$$2k = a^2 \Rightarrow 2 = \frac{a^2}{k}$$

Also $\frac{4}{3} = \int_0^a \left[x \cdot \left(\frac{x}{k}\right)\right] dx$ ✓

$$\frac{4}{3} = \int_0^a \left(\frac{x^2}{k}\right) dx$$

$$\frac{4}{3} = \left(\frac{x^3}{3k}\right)\Big]_0^a$$

$$\frac{4}{\cancel{3}} = \frac{a^3}{\cancel{3}k} \checkmark$$

$$4 = \frac{a^2}{k} \cdot a$$

$$4 = 2 \cdot a$$

$$a = 2 \checkmark$$

$$\therefore 2k = a^2$$

$$2k = 4$$

$$k = 2 \checkmark$$

$\boxed{6}$

End of Questions for Booklet One

Spare Working Page

Spare Working Page

Spare Working Page